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AN. ELECTRICAL APPARATUS FOR SOLVING HOMOGENEOUS AND NONHOMOGENEOUS
 ORDINARY LINEAR DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS OF
 HIGH ORDER, WHICH GIVES THE SOLUTION IN THE FORM OF TAYLOR'S SERIES

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[Figures are appended.]

If there is given a differential equation of the n-th order with constant coefficients:

$$a_0 y + a_1 y' + a_2 y'' + \dots + a_n y^{(n)} = f(t), \quad (1)$$

for which it is necessary to find a solution having the initial conditions:

$$\text{for } t = \tau, y = Y_0, y' = Y_1, \dots, y^{(n-1)} = Y_{n-1}, \quad (2)$$

then, in the case where all the derivatives of the sought-for function are continuous functions of t in the interval from τ to T, the solution of equation (1) in this interval can be represented in the form of a series:

$$y = Y_0 + \frac{Y_1}{1!}(t-\tau) + \frac{Y_2}{2!}(t-\tau)^2 + \dots + \frac{Y_m}{m!}(t-\tau)^m + \dots \quad (3)$$

Thus, to write the solution in series form, it is necessary to find the values of the coefficients $Y_0, Y_1, Y_2, \dots, Y_m, \dots$

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To find these coefficients we find the corresponding derivatives of y with respect to t from series (3); therefore, we obtain:

$$\begin{aligned} y &= Y_0 + \frac{Y_1}{1!} (t-\tau) + \frac{Y_2}{2!} (t-\tau)^2 + \dots + \frac{Y_m}{m!} (t-\tau)^m + \dots \\ y' &= Y_1 + \frac{Y_2}{1!} (t-\tau) + \frac{Y_3}{2!} (t-\tau)^2 + \dots + \frac{Y_{m+1}}{m!} (t-\tau)^m + \dots \\ y^{(k)} &= Y_k + \frac{Y_{k+1}}{1!} (t-\tau) + \frac{Y_{k+2}}{2!} (t-\tau)^2 + \dots + \frac{Y_{k+m}}{m!} (t-\tau)^m + \dots \end{aligned} \quad (4)$$

We expand further the right side of equation (1) into a power-series in $(t-\tau)$:

$$a_0 y + a_1 y' + a_2 y'' + \dots + a_n y^{(n)} = f(\tau) + \frac{f'(\tau)}{1!} (t-\tau) + \frac{f''(\tau)}{2!} (t-\tau)^2 + \dots \quad (5)$$

Substituting in the left side of equation (5) the values of all the derivatives from equations (4) and equating the coefficients with the same powers of $(t-\tau)$, we obtain:

$$\begin{aligned} a_0 Y_0 + a_1 Y_1 + a_2 Y_2 + \dots + a_n Y_n &= f(\tau), \\ a_0 Y_1 + a_1 Y_2 + a_2 Y_3 + \dots + a_n Y_{n+1} &= f'(\tau), \\ a_0 Y_2 + a_1 Y_3 + a_2 Y_4 + \dots + a_n Y_{n+2} &= f''(\tau), \\ &\dots \dots \dots \\ a_0 Y_k + a_1 Y_{k+1} + a_2 Y_{k+2} + \dots + a_n Y_{k+n} &= f^{(k)}(\tau), \\ &\dots \dots \dots \end{aligned} \quad (6)$$

Substituting the values of $Y_0, Y_1, Y_2, \dots, Y_{n-1}$ in the first of the equations of system (6), we can find from the initial conditions the values:

$$Y_n = \frac{1}{a_n} [f(\tau) - (a_0 Y_0 + a_1 Y_1 + a_2 Y_2 + \dots + a_{n-1} Y_{n-1})]. \quad (7)$$

Taking Y_n from equation (7) and submitting it in the second equation of system (6), we obtain Y_{n+1} and so on. [Y_0, Y_1, \dots up to Y_{n-1} were originally given, but not Y_n, Y_{n+1} ; these latter are to be determined.] It is possible to find as many coefficients of the series as desired by solving successively the following equations:

$$\begin{aligned} Y_n &= \frac{1}{a_n} [f(\tau) - (a_0 Y_0 + a_1 Y_1 + a_2 Y_2 + \dots + a_{n-1} Y_{n-1})], \\ Y_{n+1} &= \frac{1}{a_{n+1}} [f'(\tau) - (a_0 Y_1 + a_1 Y_2 + a_2 Y_3 + \dots + a_{n-1} Y_n)], \\ &\dots \dots \dots \\ Y_{n+k} &= \frac{1}{a_{n+k}} [f^{(k)}(\tau) - (a_0 Y_k + a_1 Y_{k+1} + a_2 Y_{k+2} + \dots + a_{n-1} Y_{k+n-1})]. \end{aligned} \quad (8)$$

Scheme of the Apparatus in Principle

The scheme of the apparatus in principle is designed to solve the system of equations in (8). The basic element used in the scheme is the multiplier, or multiplier, which was described by the author in his article: "A New Electrical Apparatus for Harmonic Analysis and Synthesis," in Izvestiya Akademii Nauk SSSR, Otdeleniye Tekhnicheskikh Nauk, No 3, 1946.

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The general scheme of construction is shown in Figure 1. The entire construction consists of a series of multipliers from N_0 to N_{n-1} (where n is the order of the differential equation). The transformer of each multiplier feeds a horizontal voltage divider, for which the following voltages have been fixed:

$$u_0 = -u \frac{a_0}{a_n}, \quad u_1 = -u \frac{a_1}{a_n}, \quad \dots, \quad u_{n-1} = -u \frac{a_{n-1}}{a_n}, \quad (9)$$

where u is the total voltage applied to the terminal of the horizontal divider.

The vertical dividers are fed from the horizontal dividers, according to the scheme in Figure 1. For these the following voltages are fixed by means of slides:

$$\begin{aligned} v_0 &= u_0 Y_0, & v_1 &= u_1 Y_1, & v_2 &= u_2 Y_2, & \dots, & v_{n-1} &= u_{n-1} Y_{n-1} \\ \text{or } v_0 &= -u \frac{a_0}{a_n} Y_0, & v_1 &= -u \frac{a_1}{a_n} Y_1, & \dots, & v_{n-1} &= -u \frac{a_{n-1}}{a_n} Y_{n-1}. \end{aligned} \quad (10)$$

After this, in a similar manner, the following voltage is fixed for the divider of transformer A: $U = u \frac{f(a)}{a_n}$.

When the setup is as indicated in the scheme, all voltages v will be added; as a result the following voltage will be established between the terminals r and l :

$$v_s = u \frac{1}{a_n} \left[f(a) - (a_0 Y_0 + a_1 Y_1 + a_2 Y_2 + \dots + a_{n-1} Y_{n-1}) \right], \quad (11)$$

that is, this voltage, according to equation (7), will equal:

$$v_s = u Y_n. \quad (12)$$

If the voltage of transformer B equals u , then by sliding the runner, with respect to the vertical divider of this voltage and by observing the galvanometer, we can compensate for v_s (during compensation the needle of the galvanometer will stand at zero).

In this way, if the scale of the vertical potentiometers is graduated in sections from the voltage applied to it, then one can directly read off the value of Y_n on the scale of the potentiometer of transformer B. From this point on, the operation of solving a differential equation comes to the following procedure:

By the use of a ring commutator and the turning of its crank to one division, all vertical potentiometers are shifted one to the left; that is, the potentiometer 0 is connected to transformer B, potentiometer 1 to transformer 0, potentiometer 2 to transformer 1 and so on.

If one now fixes in transformer A the following voltage:

$$U = u \frac{f'(a)}{a_n}, \quad (13)$$

then, in the next vertical potentiometer connected to transformer B, it is possible to measure the value of Y_{n+1} ; here, the process of measurement will be simultaneous with the process of fixing Y_{n+1} for further measurements.

By carrying out further ring shifts of the vertical potentiometers by means of a commutator (each time one to the left), one can obtain as great a number of the coefficients of the expansion as desired.

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The vertical and horizontal potentiometers are employed in the form expressed in Figure 2. The circular rheostat P possesses resistance equal to the resistance of one section of the resistance k. The first sign is fixed on the step rheostat k, the second and third signs on the smoothly regulable rheostat P. The total resistance between terminals a and b for the horizontal potentiometer is made 100 times less than the total resistance of the same vertical potentiometer in the scheme.

For horizontal potentiometers, terminals a and b are connected to the transformer; terminals c and d are used for feeding the vertical potentiometers.

In the vertical potentiometers terminals a and b serve for connection through the commutator to the horizontal potentiometers; terminals c and d, for the successive joining of all vertical potentiometers.

Change of sign. To change the sign of a_k (the coefficients of the derivatives in equation (1)) from plus to minus, it is necessary to provide for a switch-over device like that shown in Figure 3. A similar device is provided for changing the sign of Y_k .

Change of scale. Where the coefficients a_k of the differential equation and the values of Y_i fluctuate in a very wide interval, a device is employed for varying the scale. This device can be used in various ways for the vertical and horizontal dividers.

For the vertical dividers it is expedient, for the purpose of decreasing voltage, to connect up additional resistances, since the total resistance of the vertical divider does not vary in the process of operation with the apparatus (the divider does not carry any load). The horizontal dividers carry the vertical divider as a load; therefore, it is expedient here, for varying the scale, to make proper connection from the feeding transformer. The general scheme of the multiplier with the possibility of changing the sign of both a_k and Y_i and also their scales is shown in Figure 3.

[Appended figures follow]

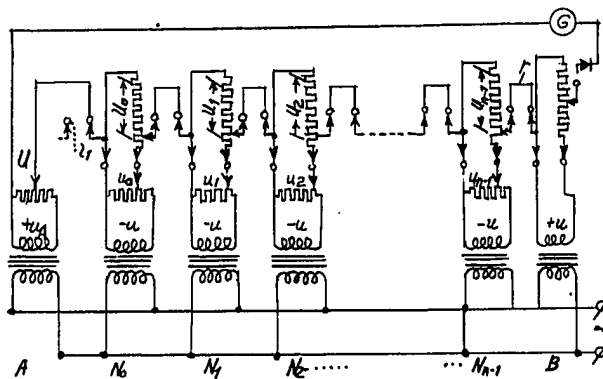


Figure 1

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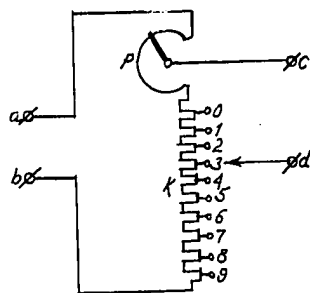


Figure 2

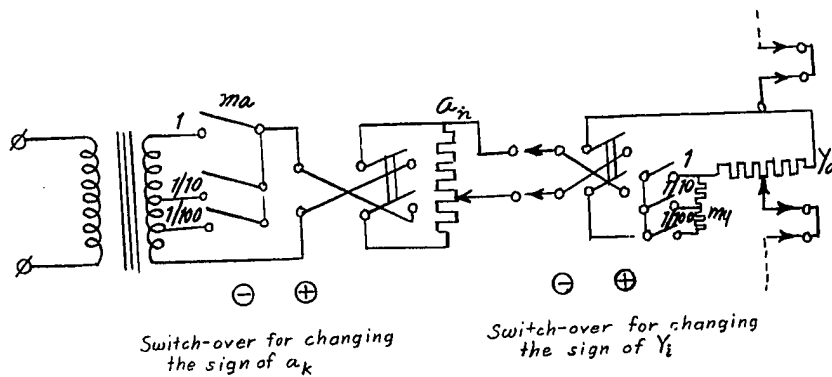


Figure 3

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